Fertigation in furrows and level furrow systems II: field experiments, model calibration and practical applications

J. Burguete[1], N. Zapata[2], P. García-Navarro[3], M. Maïkaka[4], E. Playán[5], and J. Murillo[6]

[1]: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC. P.O. Box. 13034, 50080 Zaragoza, Spain. E-mail: jburguete@eead.csic.es*

[2]: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC. P.O. Box. 13034, 50080 Zaragoza, Spain. E-mail: vzapata@eead.csic.es*

[3]: *Professor, Dept. Fluid Mechanics, Centro Politécnico Superior, University of Zaragoza. María de Luna 3, 50018 Zaragoza, Spain. E-mail: pigar@unizar.es*

[4]: *Engineer, SERS Consulting, Paseo Rosales 34, 50008 Zaragoza. Spain. E-mail: TIEMAGO@terra.es*

[5]: *Researcher, Dept. Suelo y Agua, Estación Experimental de Aula Dei, CSIC. P.O. Box. 13034, 50080 Zaragoza, Spain. E-mail: playan@eead.csic.es*

[6]: *Assistant Professor, Dept. Fluid Mechanics, Centro Politécnico Superior, University of Zaragoza. María de Luna 3, 50018 Zaragoza, Spain. E-mail: Javier.Murillo@unizar.es*

Abstract

Furrow fertigation can be an interesting practice when compared to traditional overland fertilizer application. In the first paper of this series, a model for furrow fertigation was presented. The simulation model combined overland water flow (Saint-Venant equations), solute transport (advection-dispersion) and infiltration. Particular attention was paid to the treatment of junctions present in level furrow systems. In this paper, the proposed model is validated using five furrow fertigation evaluations differing in irrigation discharge, fertilizer application timing and furrow geometry. Model parameters for infiltration and roughness were estimated using error minimization techniques. The error norm was based on observed and simulated values of advance time, flow depth and fertilizer concentration. Model parameters could be adequately predicted from just one discharge experiment, although the use of more experiments resulted in decreased error. The validated model was applied to the simulation of a level furrow system from the literature. The model adequately reproduced irrigation advance and flow depth. Fertigation events differing in application timing were simulated to identify conditions leading to adequate fertilizer uniformity.

Keywords: Infiltration, Furrow irrigation, Surface irrigation, Shallow water, Flow simulation, Numerical models.

INTRODUCTION

A computational model for furrow fertigation was presented and analyzed in a companion paper. The model introduces optimal treatment of the junctions for water and solute transport. It was successfully applied to the simulation of steady and unsteady flow with solute transport in channels and channel junctions.

Water and fertilizer uniformity and efficiency are closely related, although fertigation practices can be identified that result in higher uniformity and efficiency for fertilizer application than for water application. Fertilizer uniformity has been addressed by a number of authors (Boldt et al. 1994; Playán and Faci 1997; García-Navarro et al. 2000; Abbasi et al. 2003; Sabillón and Merkley 2004; Zerihun et al. 2003; Strelkoff et al. 2006) generally using the concept of low-quarter distribution uniformity (Merriam and Keller 1978; Burt et al. 1997). The estimation of fertilizer distribution uniformity requires an infiltration equation, advance and recession data and intense sampling of the overland water at a number of observation stations. The very high correlation that typically relates agricultural fertilizer concentration and electric conductivity permits to estimate concentration from simple conductivity measurements (Playán and Faci 1997). Fertilizer application efficiency is difficult to estimate, since assessing fertilizer deep percolation is a much more complicated task than assessing water deep percolation. A surface fertigation model including solute transport in the soil was presented by Zerihun et al. (2005a, 2005b). Despite these developments, the complexities of the interaction between soil, water, the solute and the crop have resulted in the common use of fertilizer distribution uniformity as an indicator of fertilizer performance. The identification of optimum fertilizer application practices usually involves the selection of the starting and ending fertilizer application times that result in an optimum value of the indicator of fertilizer performance. Additional factors, such as the mass of fertilizer runoff (derived from water runoff and fertilizer concentration measurements) or a qualitative estimation of deep percolation, have been considered in the choice of adequate fertigation practices.

The purposes of this paper are: 1) to present field experiments designed to validate the proposed furrow fertigation model; 2) to validate the model under typical field conditions; 3) to propose sets of model parameters representative of the experimental case obtained from partial or complete use of the calibration information; 4) to simulate fertigation in a level furrow system; and 5) to propose fertilizer application practices for the analyzed level furrow system.

MATERIAL AND METHODS

Field description and preparation

A field experiment was conducted during the summer of 2003 at the research farm of the Agricultural and Technological Research Center of the Government of Aragón in Zaragoza, Spain. The farm was located near the Gállego River. The soil was classified as typical xeroflunvent, coarse loam, mixed (calcareous), mesic (Soil Survey Staff 1992), and was formed from alluvial deposits.

Five isolated furrows were built using the same field machinery (Figure 1). One of them was connected to a secondary furrow through a T-junction. The furrow cross-sectional geometry was measured at three locations (at the upstream end, at the middle and at the downstream end) of each furrow. The average furrow dimensions were: base width $0.14m$, top width $0.80m$ and furrow depth $0.27m$. The coefficients of variation of the measurements were 10%, 7% and 10% for the base width, the top width and the furrow depth, respectively. The quality of land leveling, expressed as the standard deviation of soil surface elevation at the bottom of the furrow (Playán et al. 1996), was surveyed for each furrow with a radiometric total station. All furrows showed zero slope. The standard deviation of soil surface elevation ranged between $12mm$ at furrow Q1 and $24mm$ at furrow Q3. These values reveal that laser guided leveling was recently used in the experimental plot, and can be considered adequate for surface irrigation.

Water was diverted to each furrow using a pump. A volumetric water meter was installed downstream from the pump in order to verify that a constant discharge was applied to each furrow. An irrigation evaluation, under free draining downstream conditions, was performed at each furrow. Evaluations were characterized by different irrigation discharges: 1, 2, 3 and $4L/s$. Experiments were identified by their discharge as Q1, Q2, Q3 and Q4.

Furrow irrigation evaluations

The monitored furrow length was $100m$. The furrow spacing was $1m$. Stations were marked along each furrow every $10m$. All stations were used to monitor the advance phase (advance stations). Five of them, every $20m$, were additionally used to monitor fertilizer hydrodynamics (fertilizer stations). The fertilizer gauging points will be denoted, according to their distance to the inlet, S20, S40, S60 and S80, corresponding to distances of 20, 40, 60 and $80m$ respectively. A staff gauge was installed $2-3m$ downstream from the water inlet of each furrow to measure flow depth evolution during the evaluations. Flow depths were recorded every $2min$ for the first minutes of the experiment, and then every $5min$.

The advance times could be easily and accurately measured. However, the upstream water depth measurement met several difficulties. With flow depths in the range of $50-100mm$ and typical soil roughness values of about $20mm$, it is reasonable to assume that water flow and upstream water depth were highly affected by microtopography. Furthermore, the proximity of the inlet water pump was responsible for non-negligible oscillations in flow depth. Finally, the furrow cross section was only measured before the irrigation event, thus neglecting the changes in furrow geometry and roughness (and therefore in flow depth) induced by water flow. Upstream depth measurements constitute however an essential part of field data during the calibration process.

Fertilizer hydrodynamic evaluation

Fertilizer was applied to the irrigation water in all evaluations. The solid commercial fertilizer 12:9:34 was used because of its high water solubility and because its concentration was highly correlated with the electrical conductivity ($EC, dS/m$) of the fertilized water. We obtained the following relationship between electrical conductivity and fertilizer concentration ($s, g/L$):

$$s = 1.01EC - 2.15 \tag{1}$$

with a coefficient of determination of 0.999. The initial fertilizer concentration (approximately $10.6g/L$) was kept constant for all furrow irrigation events. This concentration value was set to ensure adequate fertilizer detection and to avoid precipitation problems. The fertilizer weight corresponding to each experimental furrow discharge was divided in 30 doses, which were previously prepared at the laboratory. Fertilizer application started in each evaluation when the advance front reached a distance of $30m$ from the inlet, and lasted for $5min$. The doses were hand applied at a constant rate, one each $10s$. A $0.5m$ furrow length located just upstream from the experiment was protected by a plastic film to facilitate fertilizer application and dissolution.

Fertilizer concentration monitoring was performed at each fertilizer station by taking water samples for laboratory analysis. Irrigation water sampling started at each station upon arrival of the fertilizer front. The location of the fertilizer front was monitorized using a field electric conductimeter (García-Navarro et al. 2000). Water samples were taken approximately every $10s$ until the fertilizer front over-passed the fertilizer station. Samples were kept in plastic bottles that were identified with the experiment discharge, the fertilizer station point and the sampling time. The electric conductivity of the water samples was determined at the laboratory using a precision electric conductimeter. The relationship between electric conductivity and fertilizer concentration presented in (1) was used to estimate the spatial and temporal variability of solute concentration in the irrigation water.

Calibration of the model empirical coefficients

Experimental data are required to calibrate the empirical roughness and infiltration coefficients included in the mathematical model. During the calibration process, the coefficients leading to a minimum difference between the numerical and experimental results are sought. In order to estimate the error that measures this difference, the following experimental variables were available: time of advance, upstream flow depth and fertilizer concentration at several stations and times.

In a first step, the error function or error norm must be defined. Assuming that N experiments have been performed and that, for each experiment j, N^j pairs of experimental measurements x_i^j and numerical simulation values X_i^j are available, the standard mean square error can be defined as:

$$E = \sqrt{\sum_{j=1}^{N} \frac{\sum_{i=1}^{N^j} (x_i^j - X_i^j)^2}{N N^j}} \tag{2}$$

This error has the dimensions of the variable x, and hence has to be made dimensionless by means of a value of x characteristic of the particular experiment if a compound error norm involving the error in several magnitudes is sought. In this case, we are interested in the best possible simulation of, simultaneously, the time of advance, the upstream water depth and the fertilizer concentration evolution. Therefore, optimal results will be obtained if the three sets of experimental data are used in the error norm definition and the subsequent parameter calibration. Let us call N the number of furrows and, for every furrow j, N_a^j the number of gauging stations for time of advance. Calling t_i^j the measured times at every gauging station i and T_i^j those obtained from the simulation, the error in advance is defined as:

$$E_a = \sqrt{\sum_{j=1}^{N} \frac{\sum_{i=1}^{N_a^j} (t_i^j - T_i^j)^2}{N N_a^j (t_{N_a^j}^j)^2}} \tag{3}$$

On the other hand, having N_c^j fertilizer concentration gauging stations in every furrow j and assuming that at every gauging station k, N_k^j concentration measurements $(s_k^j)_i$ have been obtained and there are $(S_k^j)_i$ numerical concentration values from the model, the error in concentration is defined as:

$$E_c = \sqrt{\sum_{j=1}^{N} \sum_{k=1}^{N_c^j} \frac{\sum_{i=1}^{N_k^j} [(s_k^j)_i]^2 - [(S_k^j)_i]^2}{N N_c^j N_k^j (10.6)^2}} \tag{4}$$

Where $10.6 g/L$ is the maximum experimental fertilizer concentration. If there are N_h^j upstream water depth measurements denoted h_i^j and H_i^j water depth simulated values are available, the error in water depth is defined as:

$$E_h = \sqrt{\sum_{j=1}^{N} \frac{\sum_{i=1}^{N_h^j} (h_i^j - H_i^j)^2}{N N_h^j (h_{N_h^j}^j)^2}} \tag{5}$$

The total error can be defined as:

$$E = E_a + 0.5(E_c + E_h) \tag{6}$$

The factor 0.5 is applied as an *ad hoc* weighting coefficient since, as previously indicated, the water depth experimental values contain larger experimental errors which lead to a large magnitude in the error E_h as compared to E_a (see for instance Figure 2). On the other hand, even a slight lag in the arrival of the fertilizer can induce important values of E_c. A reduction of the weight corresponding to E_h and E_c was therefore considered convenient so that all three errors contribute in a balanced way to the total error norm.

The second step in the calibration is the search for the empirical model coefficients associated to a minimum value of the error norm as defined above. Typical roughness and infiltration coefficient values as taken from the literature (Chow 1959; Soil Survey Staff 1992), were established as maximum and minimum bounding values. Next, a 10x10x10 coefficient matrix was generated. Given the computational speed of our model (around 1s per experiment in a laptop computer) a traditional algorithm based on the simulation of each of the possible coefficient combinations was chosen. This involved 1,000 coefficient sets in each furrow experiment and hence approximately one hour of computation. The process was then repeated in the neighborhood of the point associated to a minimum error and, in a couple of iterations (around three hours of computational time in total), the globally calibrated coefficient values were identified with two decimal positions of accuracy.

RESULTS AND DISCUSSION

Case I: Fertigation in furrows

Table 1 shows the values of the empirical coefficients and the error norm obtained in the calibration process. The Table presents the coefficients for the friction parameter, alternatively using the Gauckler-Manning and the friction model proposed in the companion paper. The proposed infiltration model was used in all cases. The first set involves the use of all experiments Q1-Q4. The rest of the sets is based on the data of one single experiment to estimate the friction and infiltration parameters. In each set the error norm was computed from the experimental furrow(s) data used for calibration, and the parameters were identified minimizing that error norm. For comparison purposes, the last column of the Table presents the results of the error norm determined using the whole set of experiments. In this way, all furrow sets can be fairly compared in terms of the error produced in the simulation.

The adopted calibration model involved furrows Q1-Q4, since this set showed the lowest error for both friction models. Using only one experiment resulted in an error increment within the range 11-38% using the proposed model and within the range 19-48% using the Gauckler-Manning model. This is quite an important result since it shows that the proposed model has the capacity to predict the effect of flow conditions on friction and furrow infiltration. The model benefits from detail experimentation in a range of discharges, but can produce very reasonable results by introducing data from just one discharge experiment.

The proposed friction model performs slightly better than the Gauckler-Manning model, leading to a reduction of the error by 8-18% in all calibration sets of experiments. Part of the observed error must be attributed to simplifications like those adopted to represent furrow geometry. Consequently, the error reduction due to the proposed friction model results particularly relevant.

The procedure used in this paper results in an adjustment of all model parameters to a set of irrigation experiments. Additional parameters have been used by other authors to accommodate variations in experimental conditions. Regarding infiltration, Walker and Skogerboe (1987) proposed an additional exponent to model the variation of infiltration with wetted perimeter. Playán et al. (2004) used a similar exponent applied to the variation of infiltration with discharge in a furrow section. Regarding roughness, García-Navarro et al. (2000) adjusted different values of Gauckler-Manning n in the same impervious, non erodible border when simulating unsteady advance and steady state flow. The proposed model does not require additional parameters to simulate fertilizer transport, although fertilizer concentration is used as an additional source of calibration information to estimate infiltration and roughness parameters. Owing to the parameter estimation procedure adopted in this work, the resulting parameters represent a compromise between advance, flow depth and solute transport.

Due to the characteristics of the proposed infiltration model, a pair of K and a values represent infiltration in the furrows for all evaluated discharges. Playán et al. (2004) reported values of a in the range 0.50-0.62 when irrigating similar furrows in the same experimental farm. In order to illustrate the calibration process leading to the use of experimental information from furrows Q1 to Q4, in Figure 2 the error norms (6), (3), (5) and (4) are presented in a $K - a$ domain using the proposed friction model with ($\epsilon = 0.027$). The three individual criteria lead to three different optimum values of the parameters. The global criterion provides a long, narrow region of possible values minimizing the error (from a=0.2 to 0.6, with minimum error at a=0.28). This region results similar to the one obtained using advance alone. In this particular case, advance information alone could result in a reasonable estimation of the model infiltration parameters.

In the following paragraphs simulation results are presented and compared with field data. Simulations are presented for the proposed friction model and for different variables and calibration approaches.

Figure 3 presents the comparison between experimental and simulated advance times at the different gauging stations for every experiment using for calibration experiments Q1-Q4 (above), and experiment Q3 (below). Advance was adequately simulated when all four experiments were used for calibration. Simulation results worsened when only Q3 was used for calibration, although the simulated advance was very good in three of the four experiments.

Figure 4 presents the comparison of measured and simulated upstream flow depth values for all the experiments. Simulations were performed using the parameters obtained for the calibration with the Q1-Q4 experiments. Some of the problems derived from the difficulty to obtain quality field data can be observed in this Figure. Although the expected trend of larger water depth corresponding to higher discharges was reproduced, the water depth curves corresponding to the two different discharges of experiments Q1 and Q2 resulted quite similar. The same happened between discharges Q3 and Q4. Despite the reported differences, a general agreement was observed between measured and simulated flow depth.

In Figures 5 and 6 the time evolution of measured and simulated fertilizer concentration is presented at four gauge locations for experiments Q1-Q4. All the figures display simulations performed with the set of friction and infiltration parameters described in Table 1 for the proposed friction model and using experiments Q1-Q4 for calibration. These parameters are responsible for the location of the concentration peak value. The diffusion mechanism is responsible for the amplitude and the bell shape of the concentration curves. The proposed model is free from dispersion coefficients, as it is based on Rutherford equation (see the companion paper, (Rutherford 1994)). The amplitude and location of the fertilizer concentration curves are adequately predicted by the model. In general, the fertilizer cloud travelled behind the wetting front. However, in case Q1 at gauge point S80 the model predicts that the fertilizer infiltrates much more than in the experiment. In fact, the simulated fertilizer concentration almost goes down to zero.

Case II: Irrigation and fertigation in a level furrow network

The experimental setup, described in Playán et al. (2004) and García-Navarro et al. (2004), was designed to run two separate experiments. In the first experiment, oriented to the estimation of infiltration and roughness, six irrigation evaluations were performed in level furrows (using different inflow discharges) and one irrigation evaluation was performed in a level basin. The second experiment consisted on an evaluation of level furrow irrigation performance, involving a 40-furrow setup. Since both experiments were contiguously arranged and performed virtually at the same time, the infiltration curves developed in the first experiment were considered representative of the second experiment. The parameters of the irrigation furrows were estimated using the procedures reported in the previous case (using all six furrow irrigation evaluations) as: $\epsilon = 0.28$, $l = 20mm$, $K = 2.4 \cdot 10^{-4} m/s^a$ and $a = 0.75$. For the impervious distribution channels (covered with plastic film), the Gauckler-Manning friction model was used, with the value $n = 0.02 sm^{-1/3}$ as proposed by García-Navarro et al. (2004).

Figure 7 shows the comparison between the experimental and the simulated advance time and inlet water depth at the isolated furrows. The most noticeable result is that an adequate agreement is found using a single set of parameters. Figure 8 represents the simulated location of the advancing front in the furrow network at different times. These results can be compared with a similar figure presenting the observed data in the original paper (García-Navarro et al. 2004). The observed advance is adequately reproduced in general, with specific differences which can be attributed to the field microtopography and the geometrical simplifications included in the model.

Figure 9 presents plots of measured and simulated inlet flow depth and discharge at the first furrow. Taking into account the reported experimental difficulties in the definition of the average furrow cross section and bed elevation, the agreement based on the global set of parameters can be considered very good for flow depth and approximate for discharge. Discharge at the first furrow heavily depends on the junction geometry and on the difference in roughness between the irrigation furrows and the distribution channels.

A computational experiment was performed in an attempt to demonstrate the applicability of the model. It consisted of the simulation of a fertigation event in the experimental level furrow system. This simulation was designed to assess the sensitivity of the fertilizer distribution uniformity to the fertilizer application timing. A total of $180kg$ of a hypothetical soluble fertilizer was added to the inflow discharge. Different combinations of initial and ending times of fertilizer application were considered. The distribution uniformity of water (UDW) and fertilizer (UDF) were defined as:

$$UDW = 100\frac{\overline{\alpha}_{25}}{\overline{\alpha}}, \quad UDF = 100\frac{\overline{\phi}_{25}}{\overline{\phi}}, \tag{7}$$

where α is the volume of water infiltrated per unit length of furrow, ϕ is the mass of solute infiltrated per unit length of the furrow, and $\overline{x}$ and $\overline{x}_{25}$ are the mean values of the x variable for all simulation points and for the 25% of points with lower value, respectively.

Figure 10 presents a map of UDF for a range of starting and ending fertilizer application times. Two regions could be identified as optimal for this case:

- The best option (A strategy) is the uniform and continuous application of the fertilizer from the beginning of the irrigation period almost to the cut off time (about 24 minutes), reaching a value of $UDF = 81\%$.
- The second best option (B strategy) is the sudden release of the whole amount of

fertilizer at around 7 minutes after the irrigation begins. This alternative offers slightly lower uniformity ($UDF = 78\%$), but could be more convenient for the farmer from the practical point of view.

Both strategies led to UDF higher than UDW, which attained a value of 72%. Figure 11 presents a contour plot of the distribution of infiltrated fertilizer concentration at three different times from the beginning of the irrigation event. In this case, the fertilizer was released following the timing of the B strategy. Figure 12 shows contour plots of the final infiltrated water and fertilizer per unit area. Although strategies A and B resulted in similar UDF, strategy B shows an area of very high fertilizer infiltration. This could result in a relevant loss of fertilizer to deep percolation. The application of fertilizers in a short time results in high initial concentrations. Although the processes of infiltration and dispersion can result in uniform, moderate fertilizer infiltration, there is always a risk that these high concentrations may lead to very high local fertilizer infiltration.

The conservation errors resulting from the simulation of this case II resulted similar to machine accuracy. The water conservation error was $4.0 \cdot 10^{-12}\%$, while the fertilizer conservation error was $2.1 \cdot 10^{-12}\%$ for strategy A and $2.2 \cdot 10^{-12}\%$ for strategy B. These results emphasize the adequate behavior of the proposed reach flow and junction models in what concerns to this important property. This second case study provides additional insight on the model applicability in a complex furrow network. The model can produce reliable advice on level furrow fertigation. A number of furrow experiments were used in this case to calibrate infiltration and roughness. However, the proposed model can simulate the variations in furrow infiltration and roughness as a function of the hydraulic regime from just one irrigation experiment.

CONCLUSIONS

In this paper, the numerical model presented and partially validated in the companion paper has been calibrated and validated using individual furrow and level furrow system experimental data. An error norm function was proposed to quantify the accuracy of the predicted solution and was applied to identify the set of error minimizing parameters. Only infiltration and roughness parameters were subjected to calibration, since a dispersion coefficient is not required in the proposed model.

First, a field campaign was performed to characterize solute transport in four isolated furrows during a fertigation event (case I). The model proved adequate for the prediction of both water movement and infiltration, as well as fertilizer transport. The agreement between observations and simulations relied on the quality of the information provided by the concentration gauging points, which was included in the calibration procedure.

Second, field data from a previous work was used to prove the applicability of the model in a level furrow system (case II). The optimum set of calibrated parameters was applied to all simulations in case II, leading to numerical results characterized by perfect water volume conservation and adequate agreement with the experimental data. The discrepancies between observed and predicted advance in the furrow domain were attributed to the lack of information on furrow microtopography and on the geometrical details at the junctions. Attention will have to be paid to this issue in further research. A detail characterization of furrow and junction microtopography will be required to assess model quality in a more elaborate way. Technologies such as 3D scanning will be required to produce the required topographic information.

The satisfactory model performance in fertilizer transport of case I and the reasonable ability to predict water distribution in the furrow network of case II led us to the formulation of a numerical experiment. The sensitivity of fertilizer distribution uniformity to the initial and final application times during a fertigation event in the furrow network was analyzed. Two suboptimal possibilities for fertilizer application time were identified: a) the uniform application of fertilizer during almost the full irrigation time; and b) the sudden application of the full fertilizer mass after about 7 minutes of the irrigation start. Simulations proved useful to predict the concentration distribution in time and space for all the fertilizer application possibilities. The last conclusion is related to the ability of the model to preserve the mass and solute conservation despite the presence of all the junctions that participate in the model as singular points.

References

Abbasi, F., Simunek, J., van Genuchten, M. T., Feyen, J., Adamsen, F. J., Hunsaker, D. J., Strelkoff, T., and Shouse, P. J. (2003). "Overland water flow and solute transport: model development and field-data analysis." *ASCE J. Irrig. and Drainage Eng.*, 129(2), 71–81.

Boldt, A. L., Watts, D. G., Eisenhauer, D. E., and Schepers, J. S. (1994). "Simulation of water applied nitrogen distribution under surge irrigation." *Trans. of the ASAE*, 37(4), 1157–1165.

Burt, C. M., Clemmens, A. J., Strelkoff, T. S., Solomon, K. H., Bliesner, R. D., Hardy, L. A., Howell, T. A., and Eisenhauer, D. E. (1997). "Irrigation performance measures: efficiency and uniformity." *ASCE J. the Irrig. and Drainage Div.*, 123(6), 423–442.

Chow, V. T. (1959). *Open channel hydraulics*. McGraw Hill, New York.

García-Navarro, P., Playán, E., and Zapata, N. (2000). "Solute transport modeling in overland flow applied to fertigation." *ASCE J. Irrig. and Drainage Eng.*, 126(1), 33–41.

García-Navarro, P., Sánchez, A., Clavero, N., and Playán, E. (2004). "A simulation model for level furrows II: description, validation and application." *ASCE J. Irrig. and Drainage Eng.*, 130(2), 113–121.

Merriam, J. L. and Keller, J. (1978). *Farm irrigation system evaluation: a guide for management*. Utah State University, Logan, Utah.

Playán, E. and Faci, J. M. (1997). "Border fertigation: field experiments and a simple model." *Irrig. Sci.*, 17, 163–171.

Playán, E., Faci, J. M., and Serreta, A. (1996). "Modeling microtopography in basin irrigation." *ASCE J. Irrig. and Drainage Eng.*, 122(6), 339–347.

Playán, E., Rodríguez, J. A., and García-Navarro, P. (2004). "Simulation model for level furrows. I: analysis of field experiments." *ASCE J. Irrig. and Drainage Eng.*, 130(2), 106–112.

Soil Survey Staff (1992). *Keys to soil taxonomy*. Pocahontas Press, Inc., Balcksbourg, VA, USA.

Rutherford, J. C. (1994). *River mixing*. John Wiley & Sons.

Sabillón, G. N. and Merkley, G. P. (2004). "Fertigation guidelines for furrow irrigation." *Spanish Journal of Agricultural Research*, 2, 576–587.

Strelkoff, T. S., Clemmens, A. J., and Perea-Estrada, H. (2006). "Calculation of non-reactive chemical distribution in surface fertigation." *Agric. Water Mgmt.*, 86(1–2), 93–101.

Walker, W. R. and Skogerboe, G. V. (1987). *Surface irrigation. Theory and practice.* Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

Zerihun, D., Furman, A., Warrick, A. W., and Sanchez, C. A. (2005a). "Coupled surface-subsurface solute transport model for irrigation borders and basins I: model development." *ASCE J. Irrig. and Drainage Eng.*, 131(5), 396–406.

Zerihun, D., Sanchez, C. A., Farell-Poe, K. L., Adamsen, F. J., and Hunsaker, D. J. (2003). "Performance indices for surface n fertigation." *ASCE J. Irrig. and Drainage Eng.*, 129(3), 173–183.

Zerihun, D., Sanchez, C. A., Furman, A., and Warrick, A. W. (2005b). "Coupled surface-subsurface solute transport model for irrigation borders and basins II: model evaluation." *ASCE J. Irrig. and Drainage Eng.*, 131(5), 407–419.

List of Tables

List of Figures

List of Tables

Table 1. Calibrated values of the friction and infiltration coefficients with Gauckler-Manning and proposed friction models and corresponding error norm for case I. The first column indicates the set of experiments used in the calibration. The last column indicate the error norm using experiments Q1-Q4.

Experiments	$n(s/m^{1/3})$	ϵ	$K(m/s^a)$	a	E_{Q1-Q4}
Q1-Q4	-	0.027	$3.07 \cdot 10^{-3}$	0.28	0.308
Q1-Q4	0.034	-	$3.65 \cdot 10^{-3}$	0.27	0.333
Q1	-	0.036	$7.13 \cdot 10^{-4}$	0.50	0.341
Q1	0.050	-	$8.57 \cdot 10^{-4}$	0.48	0.401
Q2	-	0.025	$5.00 \cdot 10^{-4}$	0.65	0.385
Q2	0.044	-	$5.87 \cdot 10^{-4}$	0.62	0.422
Q3	-	0.032	$1.26 \cdot 10^{-3}$	0.48	0.359
Q3	0.039	-	$1.16 \cdot 10^{-3}$	0.51	0.396
Q4	-	0.017	$1.04 \cdot 10^{-3}$	0.56	0.427
Q4	0.028	-	$7.00 \cdot 10^{-4}$	0.66	0.493

List of Figures

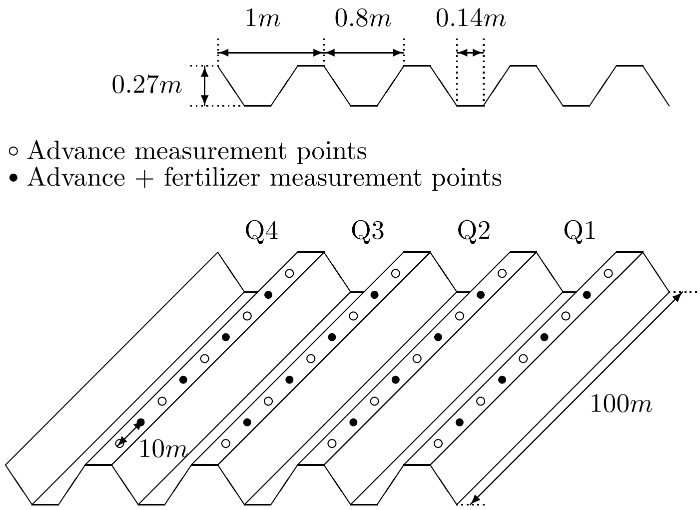

Figure 1. Experimental set-up used for furrow fertigation evaluation in case I.

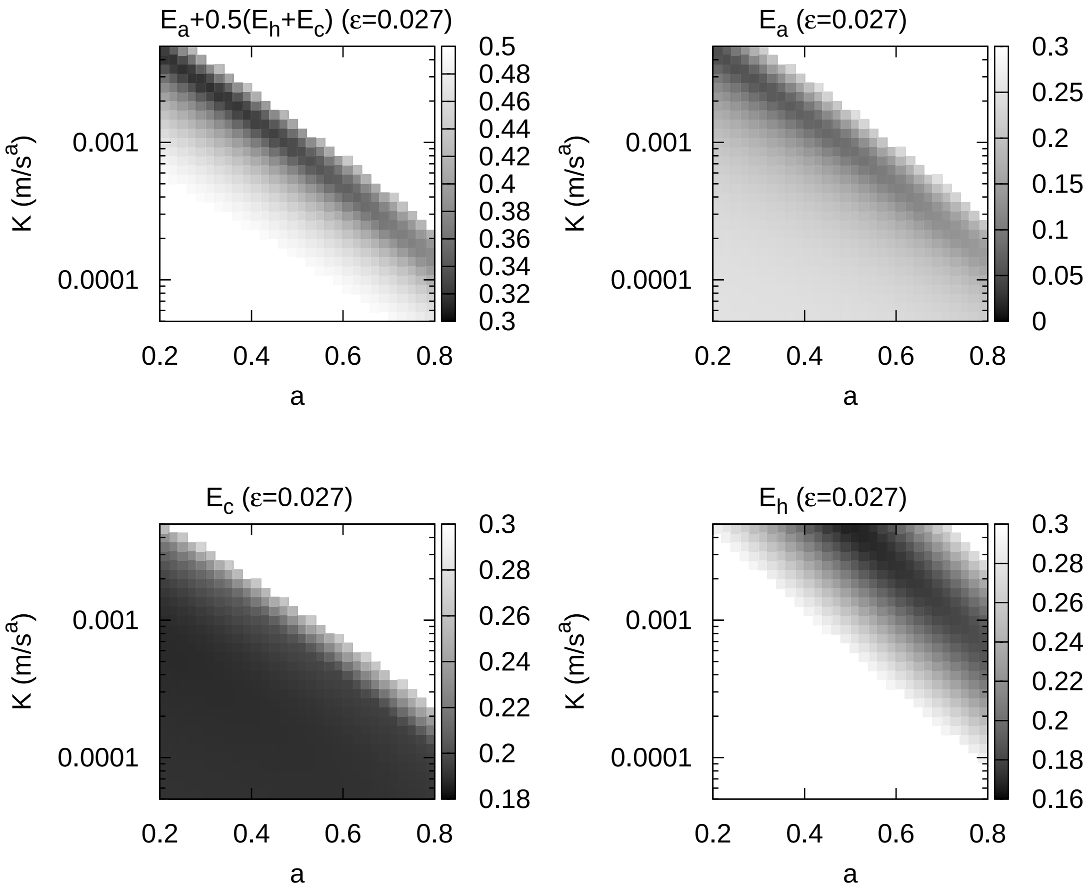

Figure 2. Different errors (E, E_a, E_c and E_h) as a function of K and a for the proposed friction model with $\epsilon = 0.027$ in case I. Experiments Q1-Q4 were used in the calibration process.

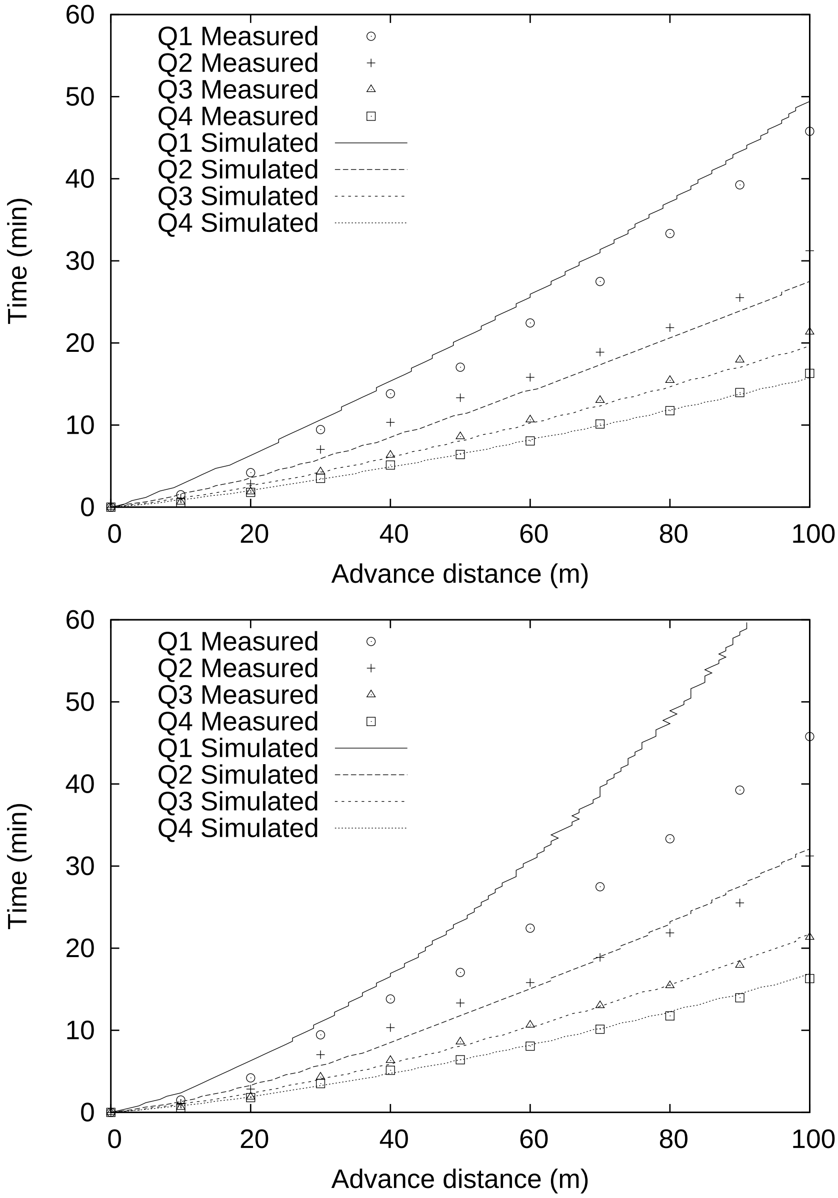

Figure 3. Advancing curves in case I using the proposed friction model with empirical parameters calibrated using the set of experiments Q1-Q4 (above) and using only experiment Q3 (below).

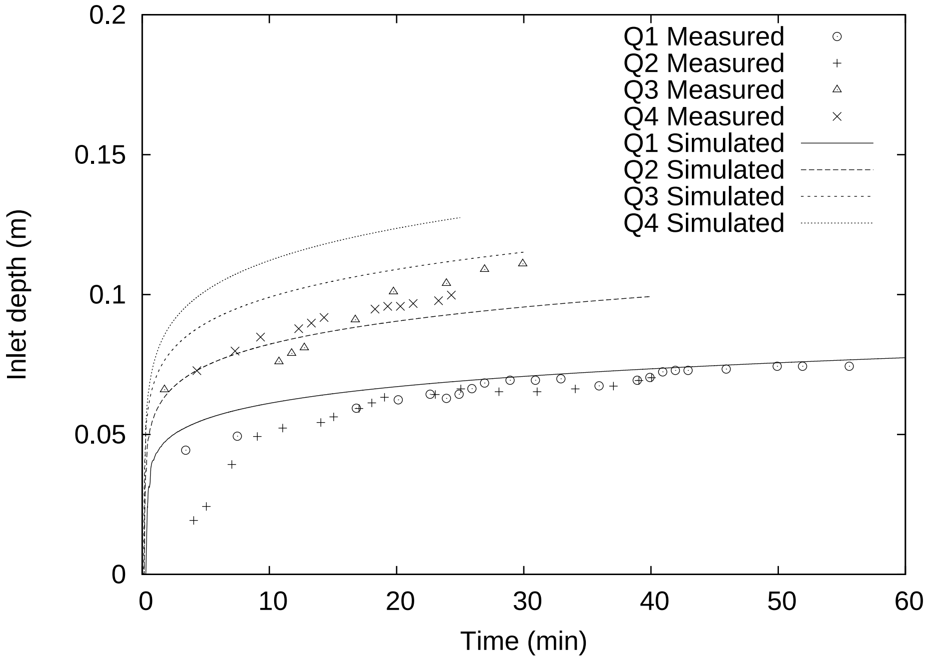

Figure 4. Upstream water depth in case I using the proposed friction model with empirical parameters calibrated using the set of experiments Q1-Q4.

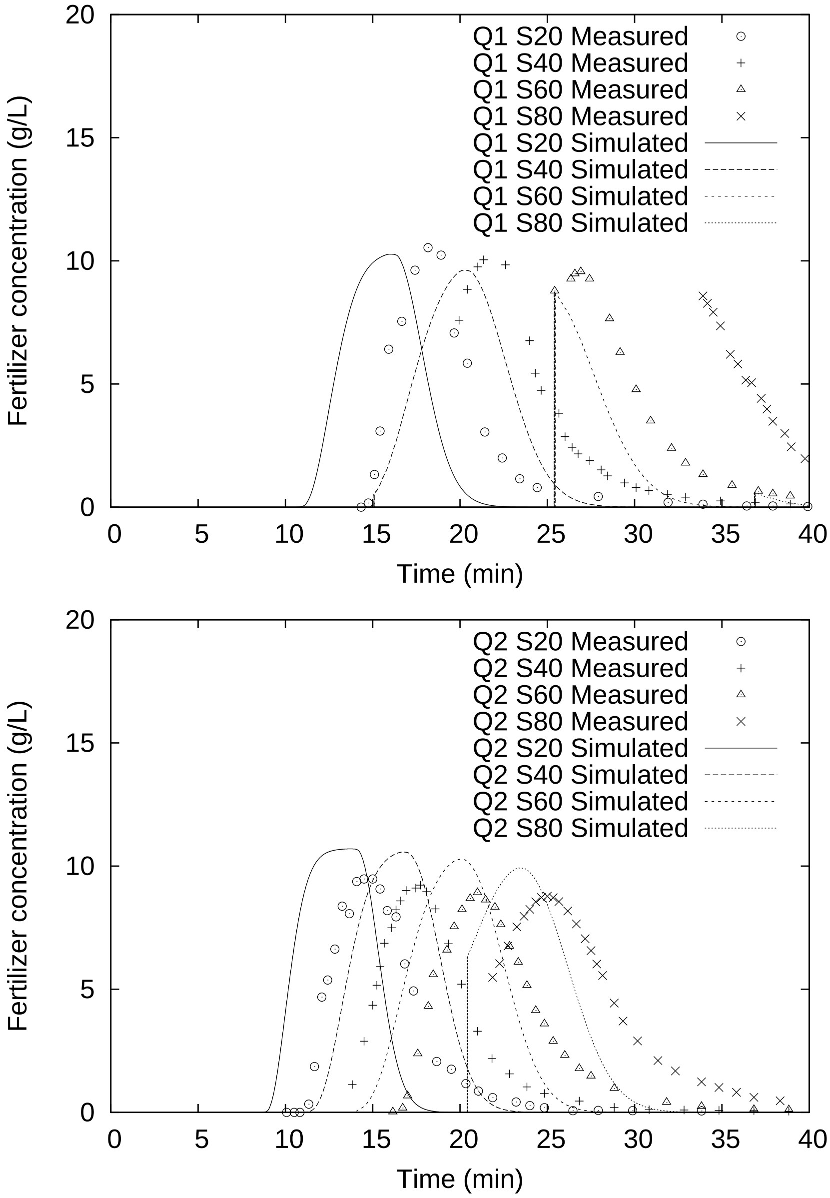

Figure 5. Fertilizer concentration in case I (experiments Q1 and Q2) using the proposed friction model with empirical parameters calibrated using the set of experiments Q1-Q4.

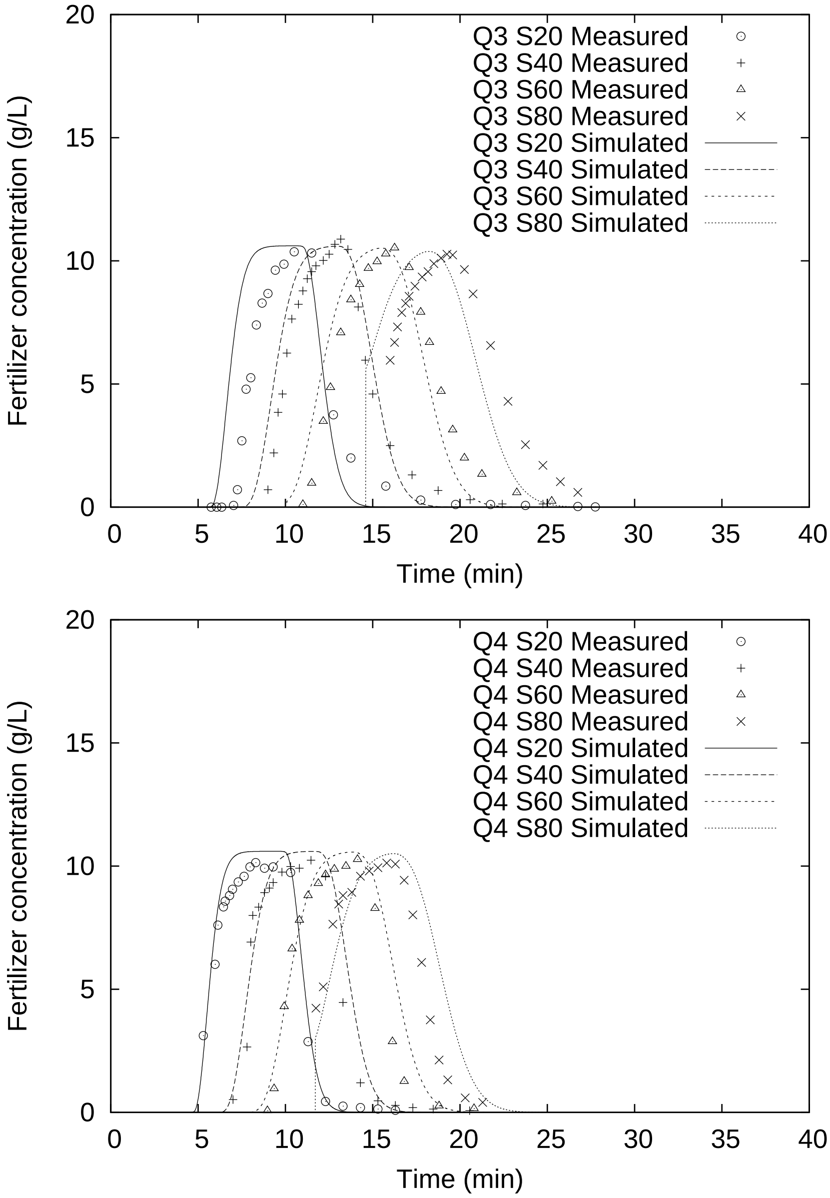

Figure 6. Fertilizer concentration in case I (experiments Q3 and Q4) using the proposed friction model with empirical parameters calibrated using the set of experiments Q1-Q4.

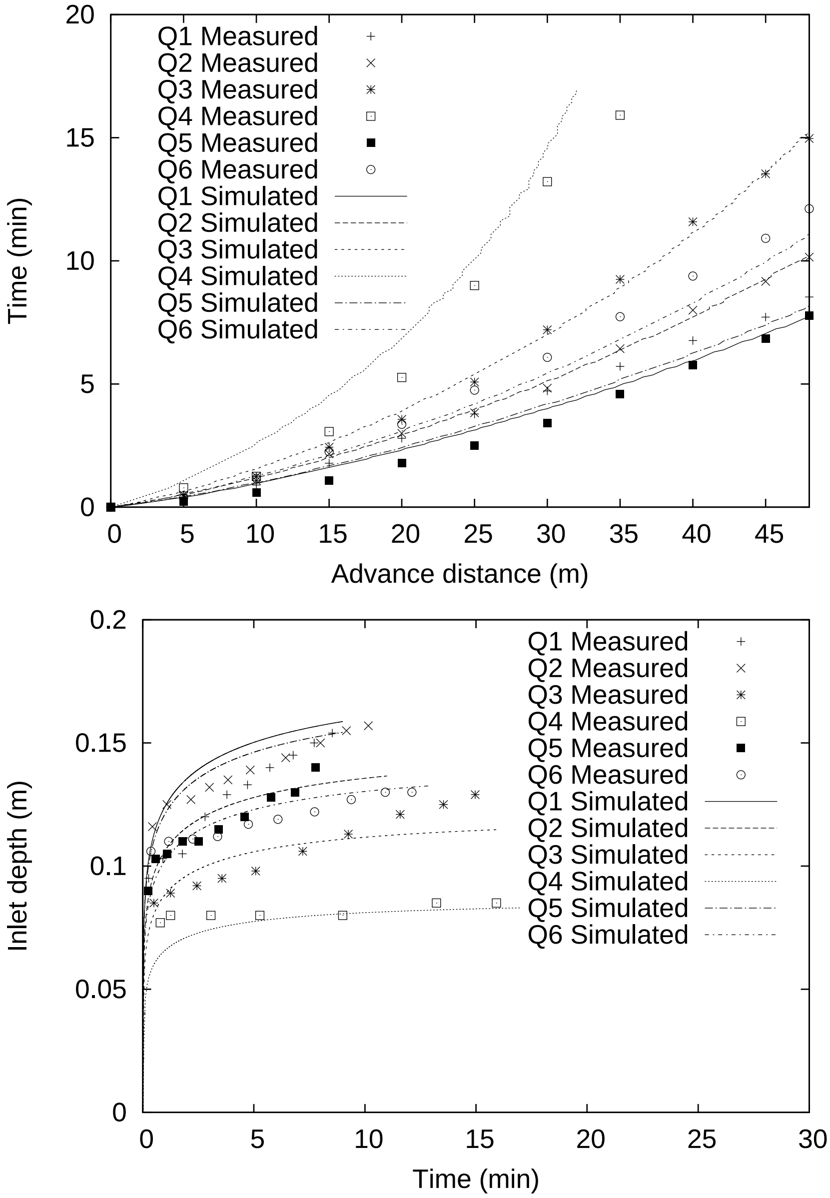

Figure 7. Advance curves and depth at inlet in the six furrows with different discharges in case II.

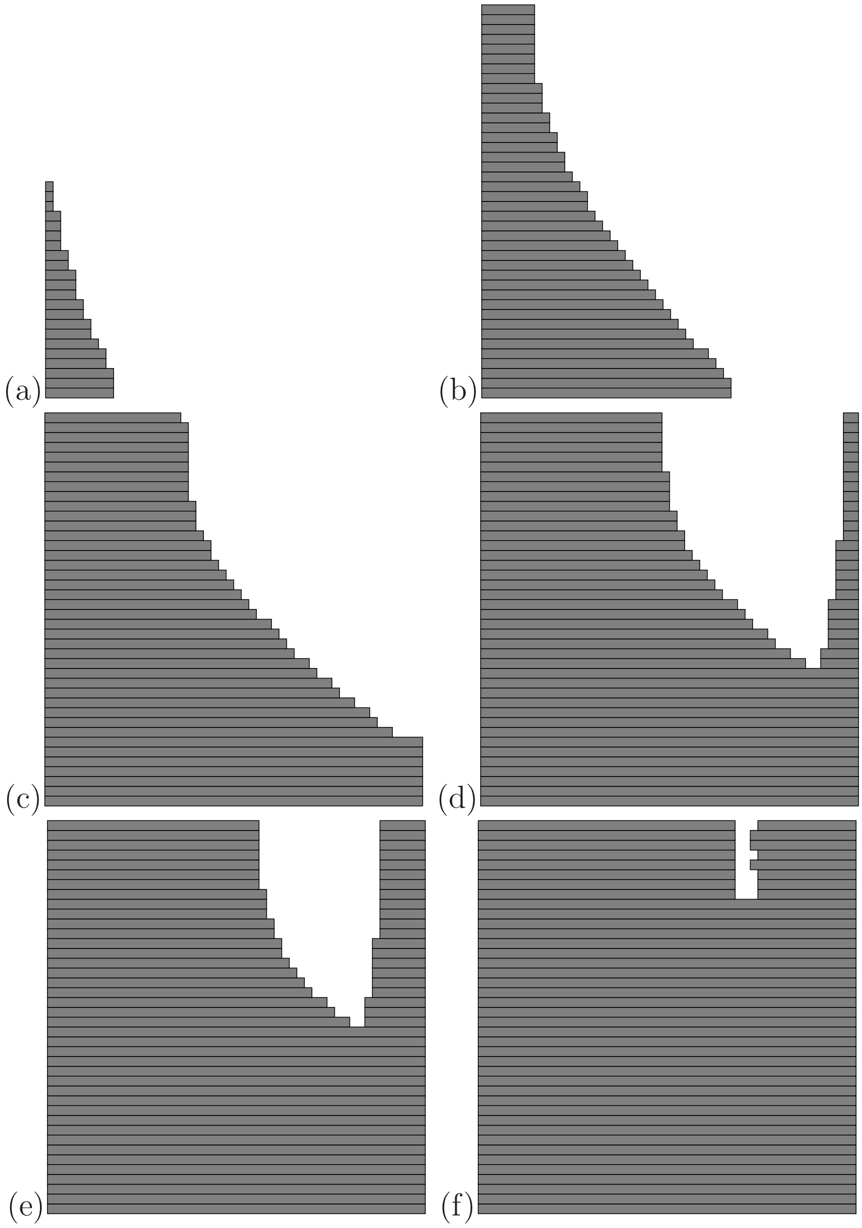

Figure 8. Map view of the simulated furrow water advance in case II for (a) $t = 1min$**, (b)** $t = 6min$**, (c)** $t = 14min$**, (d)** $t = 19min$**, (e)** $t = 23min$ **y (f)** $t = 30min$**.**

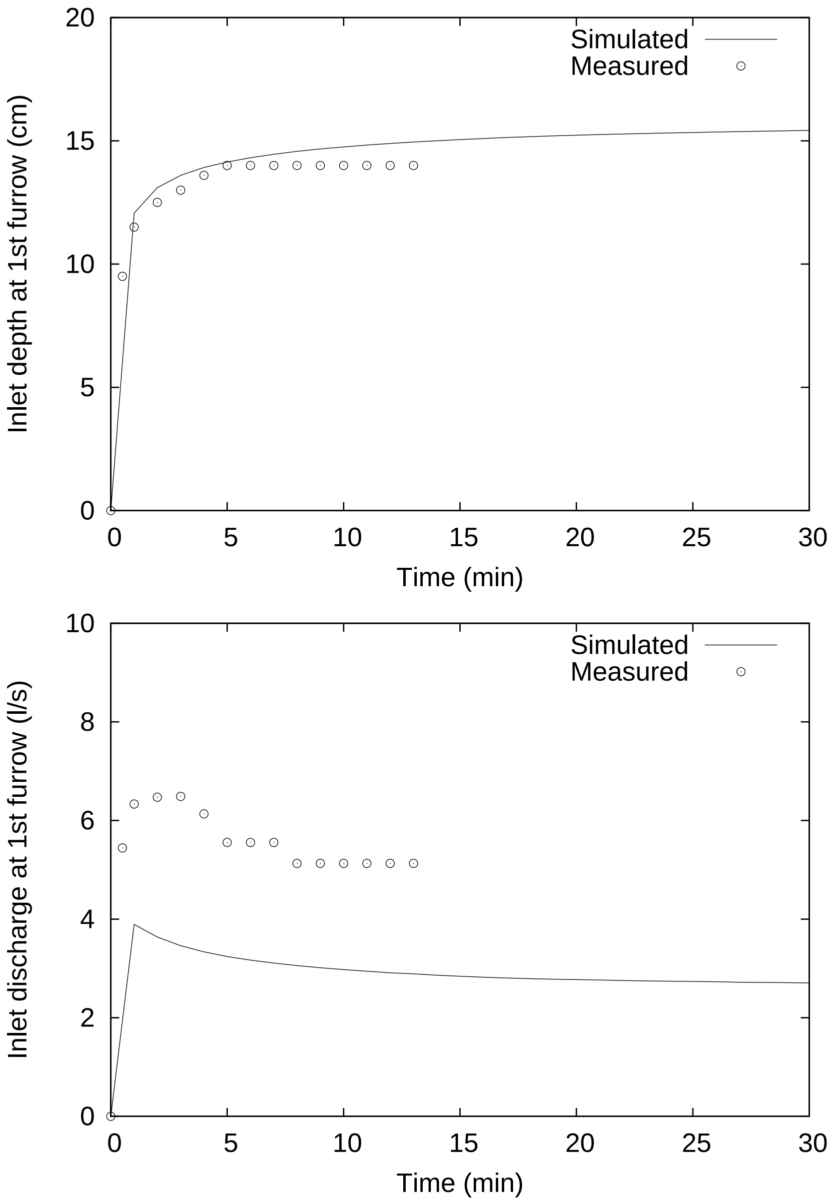

Figure 9. Inlet depth and discharge at the first furrow in case II.

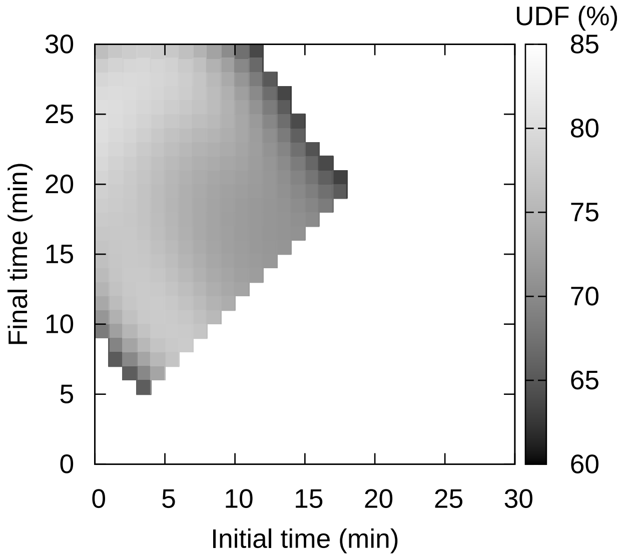

Figure 10. Uniformity of distribution of fertilizer mass in case II for different application times (not plotted regions have uniformities less than 60%).

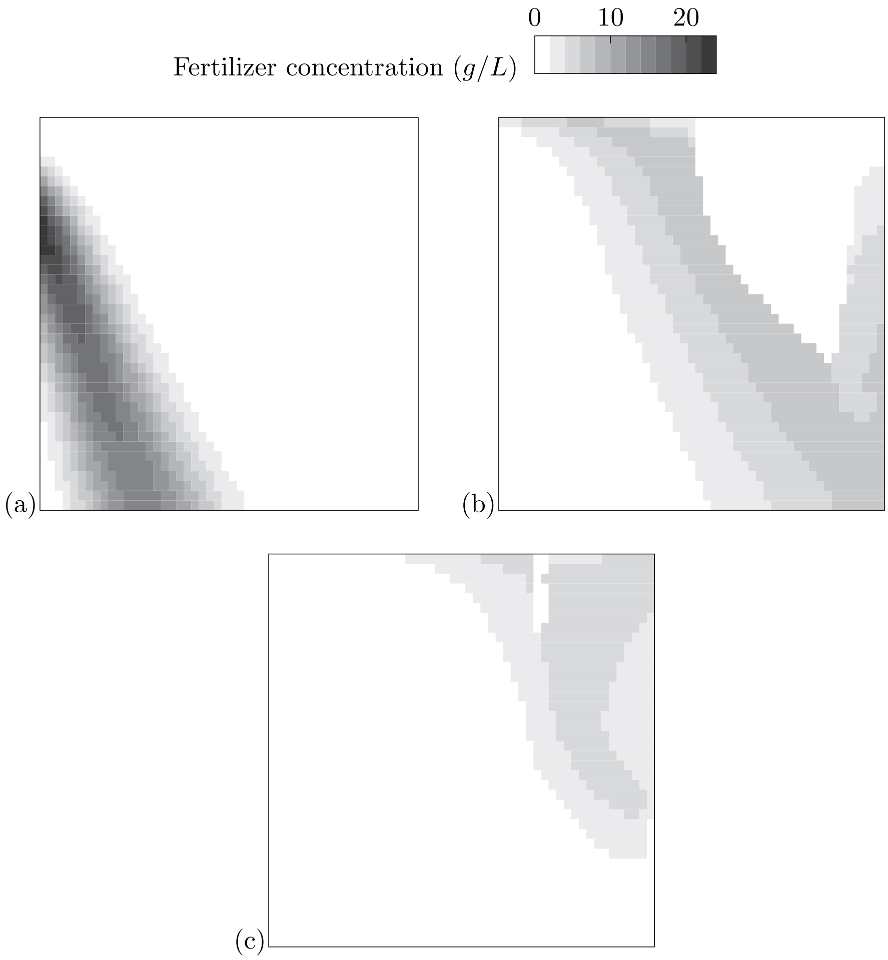

Figure 11. Map view of the surface fertilizer concentration simulated in case II with strategy B for (a) $t = 10min$**, (b)** $t = 20min$ **and (c)** $t = 30min$**.**

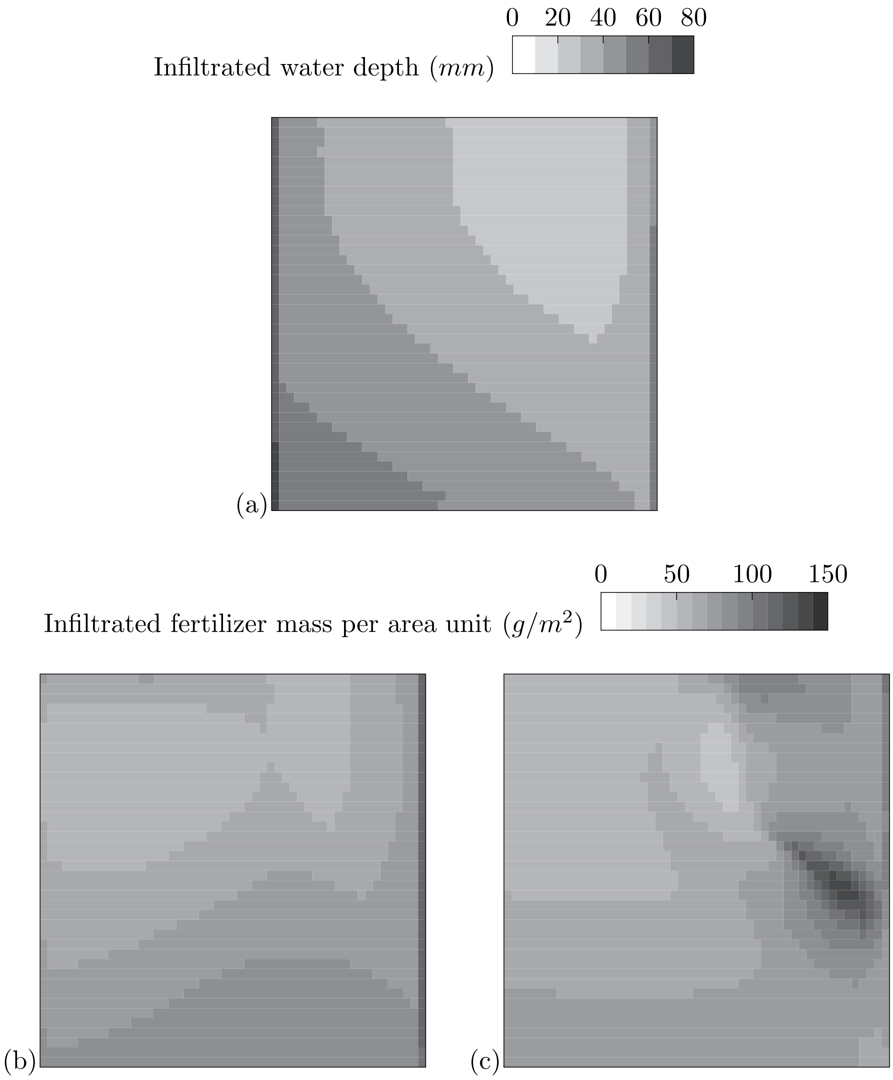

Figure 12. Map view of the infiltrated (a) water depth, (b) and (c) fertilizer mass per area unit with strategies (b) A and (c) B simulated in case II.